

# **NAVAL POSTGRADUATE SCHOOL**

## **Monterey, California**



## **THESIS**

**A SECOND LAW APPROACH TO AIRCRAFT  
CONCEPTUAL DESIGN**

by

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September 1998

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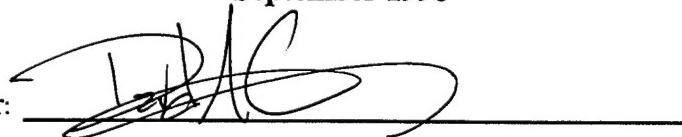
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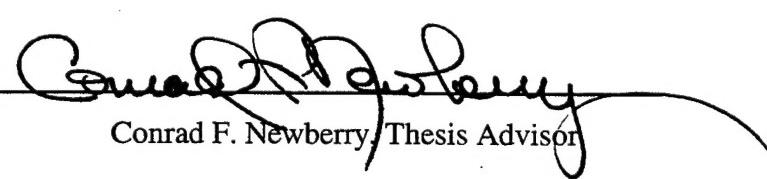
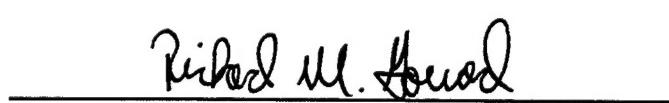
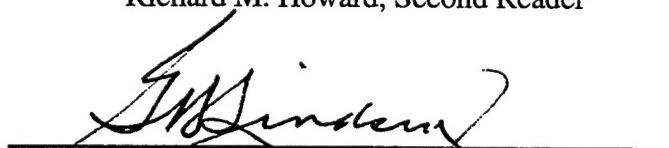
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## **ABSTRACT**

With advancements in the fields of propulsion, aerodynamics, structures, materials and controls, the routine exploration of hypersonic, atmospheric flight has become a more feasible concept. Thus, there is a need for efficient and effective hypersonic configurations. Current studies in configuration efficiency and effectiveness seem to be concentrated in aircraft subsystem design, especially propulsion systems, rather than at the conceptual aircraft system design level. This thesis attempts to initiate the process of incorporating the Second Law of Thermodynamics into the conceptual aircraft design process. The methodology for this process involves the use of the thermodynamic variable exergy, also known as availability. The ultimate goal of the process introduced by this thesis is to be able to define an aircraft configuration design space based upon both the First and Second Laws of Thermodynamics.



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## LIST OF SYMBOLS

<u>SYMBOL</u>	<u>QUANTITY</u>	<u>UNITS</u>
A	Area; availability	ft <sup>2</sup> , Btu/lbm
C <sub>D</sub>	Coefficient of drag	dim.
C <sub>D0</sub>	Coefficient of drag at zero lift	dim.
C <sub>L</sub>	Coefficient of lift	dim.
C <sub>p</sub>	Specific heat at constant pressure	dim.
D	Drag	lbf
Ex	Exergy	Btu/lbm
e	Specific energy	ft <sup>2</sup> /s <sup>2</sup>
f	Fuel-to-air weight ratio	dim.
g	Acceleration of gravity	ft/s <sup>2</sup>
H <sub>c</sub>	Heat of combustion	Btu/lbm
h	Altitude; enthalpy	ft, Btu/lbm
I	Irreversibility	Btu/lbm
J	Joule's equivalent	778 ft lbf/Btu
K	Coefficient in drag polar equation	dim.
m	Mass	lbm
P	Pressure	lbf
Q	Heat added	Btu
q	Dynamic pressure	lbf/ft <sup>2</sup> , psf

<u>SYMBOL</u>	<u>QUANTITY</u>	<u>UNITS</u>
R	Additional drag (flaps, landing gear, etc.)	lbf
S	Planform area	ft <sup>2</sup>
s	Entropy	Btu/lbm°R
T	Installed thrust	lbf
T <sub>sl</sub>	Sea level thrust	lbf
T <sub>0</sub>	Reference temperature	°R
T <sub>∞</sub>	Freestream temperature	°R
V	Velocity	ft/s
W	Instantaneous aircraft weight	lbf
W <sub>f</sub>	Weight of fuel; final aircraft weight	lbf
W <sub>i</sub>	Initial aircraft weight	lbf
W <sub>r</sub>	Fuel weight ratio	dim.
W <sub>to</sub>	Aircraft takeoff gross weight	lbf
α	Installed thrust lapse	dim.
β	Instantaneous weight fraction; fuel weight fraction	dim.
η <sub>z</sub>	Overall process efficiency	dim.
η <sub>c</sub>	Compression process efficiency	dim.
η <sub>e</sub>	Expansion process efficiency	dim.
η <sub>exergy</sub>	Exergy efficiency	dim.
η <sub>thermal</sub>	Thermal efficiency	dim.

<u>SYMBOL</u>	<u>QUANTITY</u>	<u>UNITS</u>
$\eta_p$	Propulsive efficiency	dim.
$\sigma$	Density ratio; entropy gain	dim, Btu/lbm°R
$\rho$	Density	slugs
$\rho_0$	Sea level density	slugs
$\Pi$	Throttle control setting	dim.



*"With the realization of airplane and missile speed equal to or even surpassing many times the speed of sound, thermodynamics has entered the scene and will never again be absent from our considerations."*

**J. Ackeret, 1961**

## I. INTRODUCTION

The impetus for this thesis is the desire to improve the conceptual aircraft design process. Current design methodology tends to emphasize performance parameters that can be related to the design process through an application of the First Law of Thermodynamics. A retrospective view of the aerospace industry suggests that, since December 17, 1903, improvements in aircraft capability have been continually achieved by gains in subsystem performance within this overall First Law paradigm.

Some of these improvements have been achieved by an informal paradigm of subsystem integration. Other improvements have been obtained by changing the character of the subsystem (e.g., changing from reciprocating engines and the Otto cycle to the turbojet/turbofan engines and the Brayton cycle). Additional improvements have been attained by increasing subsystem efficiencies. Subsystem efficiency improvement is often achieved by design changes based upon thermodynamic Second Law analyses.

The First Law design paradigm utilizes the concept that heat into the system or design space is converted to work, if the design process is a closed loop. The product of this design paradigm is the mathematical representation of the design space. This "First Law" design space, however, gives no particular information about the efficiency or effectiveness of a design solution within the design space. The goal of this thesis is to initiate an improvement in the conceptual design process by introducing Second Law concepts into aircraft subsystem integration through the conceptual design process. The

Second Law concepts of energy availability, or exergy, are reviewed in an attempt to identify ways in which the distribution or redistribution of energy can be used to improve aircraft system performance. This study is considered to be necessary as the aerospace industry continues its quest for higher flight speeds and increased vehicle performance.

#### A. HISTORY

From the earliest designs of aircraft – the Wright Flyer, WWI bi-planes and tri-planes – integration of an aircraft’s subsystems might be considered to have occurred on an informal basis from the perspective of a general systems theory. The importance of system integration arose as aircraft were pushed higher, faster and further. These increased capabilities were obtained through advances in propulsion technology, material science, increases in airfoil efficiency, improvements in structural design, avionics, and manufacturing processes. Throughout early aviation and into the late 1940s, these technological advancements were incorporated into the design of subsystems without too much formal regard to system integration. An example of early system integration is the F4U Corsair. The gullwing used for the Corsair arose from the need to incorporate a large powerplant and the necessarily large propeller into a carrier-based airframe where landing loads required relatively short, very strong landing gear. The size of the propeller determined the sizing of the landing gear to accommodate the necessary propeller ground clearance. Simultaneously, the landing gear loads required shorter struts, which would not accommodate the large propeller. In order to accomplish the integration of these two

requirements, a gullwing was designed that allowed shortened landing gear while providing the propeller adequate ground clearance.

This type of system integration might be considered informal in the sense that the engine seems to have been developed essentially as a separate subsystem. There seems to have been no consideration given to treating the engine and the airframe as two elements of a single system.

As aircraft and airbreathing vehicles became capable of higher speeds and altitudes, the need for system integration grew. In order to achieve a high level of integration, design methodology needed to become more formalized. This formalization was necessary to attain a vehicle design capable of meeting all of the system requirements. This process did not, necessarily, produce the best subsystems but, rather, ones that were effective in meeting the performance criteria. Steps toward formalization can be seen in the designs of the latter 1950s and throughout the 1960s. For example, the advent of supersonic flight led to improved aerodynamic configurations with such innovations as the area rule. Another example can be found in the exploration of space. The high heat loads encountered during reentry led to ablative nosecones, which enabled the vehicles and their occupants to survive. These vehicle configurations and innovations occurred through necessity and produced a more formalized process of design by considering the effects of mission requirements at the vehicle conceptual design phase rather than later during the development stage.

This formalization continued through the 1970s and into the 1980s. As aircraft were required to perform in the various regimes of flight (hypersonic, supersonic and

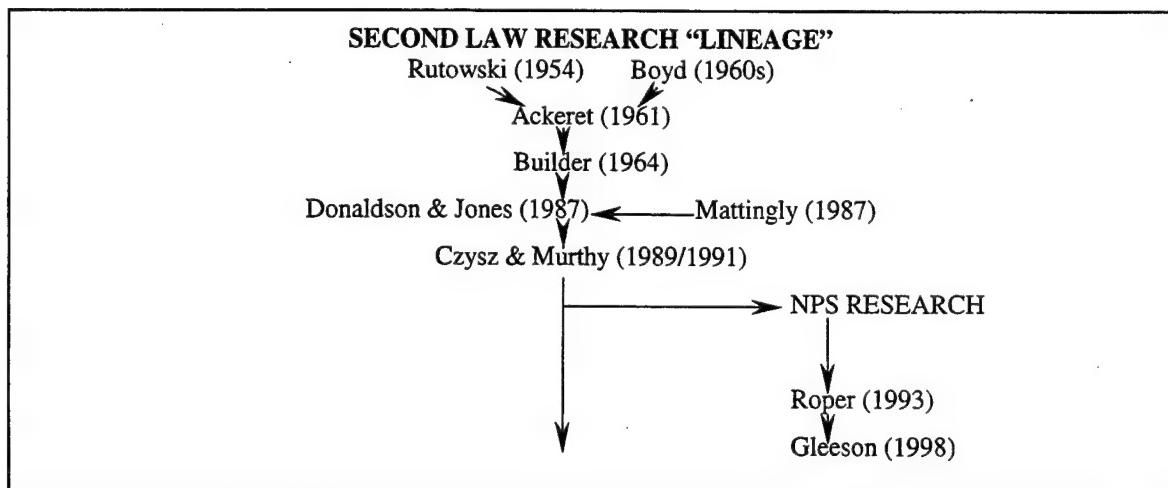
subsonic) the conceptual design phase dominated the development of innovative techniques of subsystem improvement and system integration required to achieve the design goals. Typically some 70% to 80% of the life cycle cost of an aeronautical system is determined by decisions made during the conceptual design process [Ref. 1: p. 19, Fig. 21]. A primary example of subsystem advancement and a necessarily high level of system integration is the space shuttle. The advanced vehicle requirements created a need for a high level of system integration during the conceptual design phase. For example, the aerodynamics and heat transfer of reentry had to be considered in the development of the control system. High system integration was necessarily performed at the conceptual design phase since integration during a later phase would have been impractical and cost inefficient in both weight and dollars.

Current hypersonic configuration development requires an even higher level of system integration compared to efforts in the 1980s and 1990s. Modifying any single subsystem without thought to the propagation of these changes throughout the configuration's other systems can have significant impact on system integration and, ultimately, on the configuration's performance. Consideration of waverider configuration development provides an excellent example. The aerodynamic undersurface of the vehicle will most likely serve as an inlet ramp for the propulsion system. Additionally, the propulsion system is likely to have a dramatic effect on the vehicle's wave and base drag. Modifying the propulsion system then has systemic effects on a configuration's aerodynamics, structure, control and performance. Consideration of system integration must be given during the conceptual design process for improved systemic efficiency.

## B. SECOND LAW ANALYSIS

The desire for greater speed and higher altitude has resulted in changes in the process of designing aircraft. While increases in aircraft capability can still be expected as aerodynamic, propulsion, structural, material and avionics technologies are improved, greater increases in capability may be achieved by improving the efficiencies of some of the thermodynamic processes associated with aircraft performance. These improvements may be found through a more complete thermodynamic analysis of the system. Currently, for example, configurators typically define a design space based on force balances and the first Law of Thermodynamics.

In order to consider Second Law effects in system design, an overview of design space development is helpful. As stated previously, the conceptual design process is continuing to undergo a process of formalization. Several innovations occurring during this formalization provide key insights into the basis for this thesis. The following “flowchart” emphasizes the highlights of background research used in this thesis.



Rutowski introduced one of the first concepts in energy management. His 1954 paper advanced the idea of evaluating aircraft performance based on the vehicle's specific excess energy. Evaluation of performance under certain constraints proscribed by design requirements such as turn rate and maximum speed allowed parameterization and quantification of what might be called "design space". Part of the methodology used to define the available design space is the utilization of the First Law of thermodynamics [Ref. 2: p. 188-190].

Beginning in the latter 1950s and throughout the 1960s, the ideas of energy management and evaluating an aircraft's performance based on its specific excess energy were being "cultivated" by pilots. One of the more prominent "energy-maneuverability" proponents was an Air Force pilot, Colonel John Boyd. His experiences in the Korean War led him to believe that, in air combat, the aircraft with the superior "energy state" would prevail. Through a study of the principles of thermodynamics, Boyd gained insight into the concept of specific excess energy and the role that it played in combat maneuverability. He continued his study of "energy maneuverability" throughout his flying career and taught "energy-maneuver" theories to his fellow pilots. The ideas of "energy maneuverability" and specific excess energy have come to be basic notions in aircraft conceptual design. This is most clearly seen in the conceptual design evolution of fighter aircraft throughout the late 1960s and 1970s with the F-16 being a premier example [Ref. 3: p. 13-15].

In 1961, Ackeret suggested that the first law evaluation did not provide a complete solution to advanced vehicle performance requirements. He stated that the

entropy gains incurred by the increased flight velocities must be considered and could be taken into account through a Second Law of thermodynamics analysis of vehicle subsystems. His examples of Second Law consideration include systemic overviews of the Carnot cycle, a wind tunnel, a shock system and a boundary layer. The Carnot cycle results show that the low temperature reservoir environment is much more critical to entropy rise in a thermodynamic system than the high temperature sink. The wind tunnel example demonstrates that the “entropy of the environment steadily rises” [Ref. 4: p. 83]. These examples are used to demonstrate the effectiveness of Second Law analysis in determining which subsystem or process is causing the systemic entropy rise.

In 1964, Carl Builder considered a Second Law analysis of the Brayton cycle. Builder’s analysis demonstrates that mechanical compression (as performed in the mid-60s) works well at “slow flight speeds around 1,500 feet per second.” [Ref. 5: p. 1] At higher speeds, Builder suggests that bringing the fluid to stagnation enthalpy conditions during compression (i.e., with corresponding subsonic compression) only serves to increase entropy and drives the thermal efficiency to lower values. If compression cycles can be maintained at an “optimum” level throughout flight speed variations, the overall efficiency is maintained. This efficiency even increases with flight velocity and the Brayton cycle efficiency ( $\eta_{\Sigma} = \eta_c \eta_e$ ) need not be pushed higher than 85%. Builder’s work shows that a Second Law analysis of an open-cycle provides pertinent information concerning the “entropy-producers” and that cycle efficiencies have finite values, which need not be 100% in order to produce systemic improvements. [Ref. 5: p. 2-6]

In the latter 1980's, aerospace engineers were considering the construction of a single-stage-to-orbit (SSTO), airbreathing vehicle. In 1987, Donaldson and Jones proposed that SSTO efficiency might be accomplished through better integration of powerplant and airframe. The goal of their work is to attain escape velocity by means of propulsive efficiency gains. Their analysis begins by stating that the change in kinetic energy of a high Mach number vehicle is accomplished by optimizing the thrust-to-drag ratio. Their qualitative analysis led to an "optimum" thrust-to-drag ratio of approximately 3.5. Increasing the thrust-to-drag ratio above this value provides little benefit. The analysis also suggests that modern system specific-energy-averaged propulsive efficiencies hover around 0.4. The conclusion of their discussion is that managing the vehicle fuel weight fraction offers the only realistic, controllable variable for achieving necessary escape velocity. The Donaldson-Jones analysis heightens the notion that system integration and the formalization of the conceptual design phase are necessary to achieve advances in hypersonic flight [Ref. 6: p. 32-34].

Further work in the area of second law propulsion analysis has been performed by Czysz and Murthy and presented in their papers of 1989 and 1991 delivered at AIAA conferences. The 1989 paper includes an energy availability study of a turbojet. They show that recapturing aerodynamic heat for reuse (e.g., to preheat fuel) may increase propulsive efficiency (many liquid rocket systems use hot exhaust gas in the nozzle to preheat fuel while the fuel cools the nozzle to achieve a similar result). Their methodology for examining the turbojet involved the thermodynamic variable known as exergy [Ref. 7: p. 4-8]. The 1989 work of these authors is extended in their 1991 paper.

In 1991 they dispute the statement, “energy availability studies are useful only in the initial design process.” Czysz and Murthy propose that subsystem, or “downstream”, availability analyses are also worthwhile and effective [Ref. 8: p. 2-5].

Since the goal of this thesis is to improve the conceptual aircraft design process through the incorporation of Second Law analysis in the definition of a vehicle design space, we do not dispute the viability of “downstream” analyses. However, we do agree that the concepts of energy availability and exergy are useful in determining a configuration’s effectiveness and efficiency, and the evaluation of effectiveness and efficiency is desirable at all flight speeds but very necessary at hypersonic flight speeds.



*"In the design of a new system involving the generation or use of energy, the exergy method will provide the information to better select the component designs and operation procedures that will be most effective..."*

**J. Ahern, 1980**



## II. BACKGROUND CONCEPTS

This chapter is devoted to the development of a few of the ideas and energy concepts related to thermodynamic issues in design. The major topics covered in this chapter include the development of the conceptual design space, exergy and availability.

### A. DESIGN SPACE

A fully developed design space is an  $n$ -dimensional solution space, where ( $n$ ) is the number of system parameters, for a conceptual configuration, which meets all of the individual system requirements. The creation of an  $n$ -dimensional design space is a complex concept that presents a difficult problem in determining the necessary system parameters and graphically representing the space. Therefore, a simplified version of the design space and a process for its construction and graphical representation is desired.

Loftin and Mattingly define a simplified, two-dimensional design space for use in the conceptual design phase of a vehicle [Ref. 9: p. 144-148][Ref. 10: p. 17-22]. The methodology of constructing this design space is based on force balances (i.e., steady level flight) and the First Law of thermodynamics. The general expression, used in the formulation of both Loftin's and Mattingly's design space, is in terms of the system's energy input and distribution and can be written as a rate equation [Ref. 10: p. 17]

$$\{Rate\ of\ Mechanical\ Energy\ Input\} = \quad \quad \quad (2.1)$$

$$\{Storage\ Rate\ of\ Potential\ Energy\} + \{Storage\ Rate\ of\ Kinetic\ Energy\}.$$

This statement of the energy balance can be represented in a form that allows configurators to consider the individual system requirements (e.g., turn rate, acceleration, climb performance, landing distance, etc.) and interpret them as constraints on the design space. These constraints can then be plotted in terms of system parameters, in order to generate a graphical representation of the requirements. The resulting inscribed area defines the design space. It is useful to plot these constraints on axes characterized by important design variables (e.g., wing loading ( $W_{to}/S$ ) and thrust loading, or thrust to weight ratio ( $T_{sl}/ W_{to}$ )). The area enclosed by plotting each of the constraints forms a two-dimensional design, or solution space, which satisfies all of the system requirements. Figure 1 shows the axes used in this two-dimensional formulation of the design space.

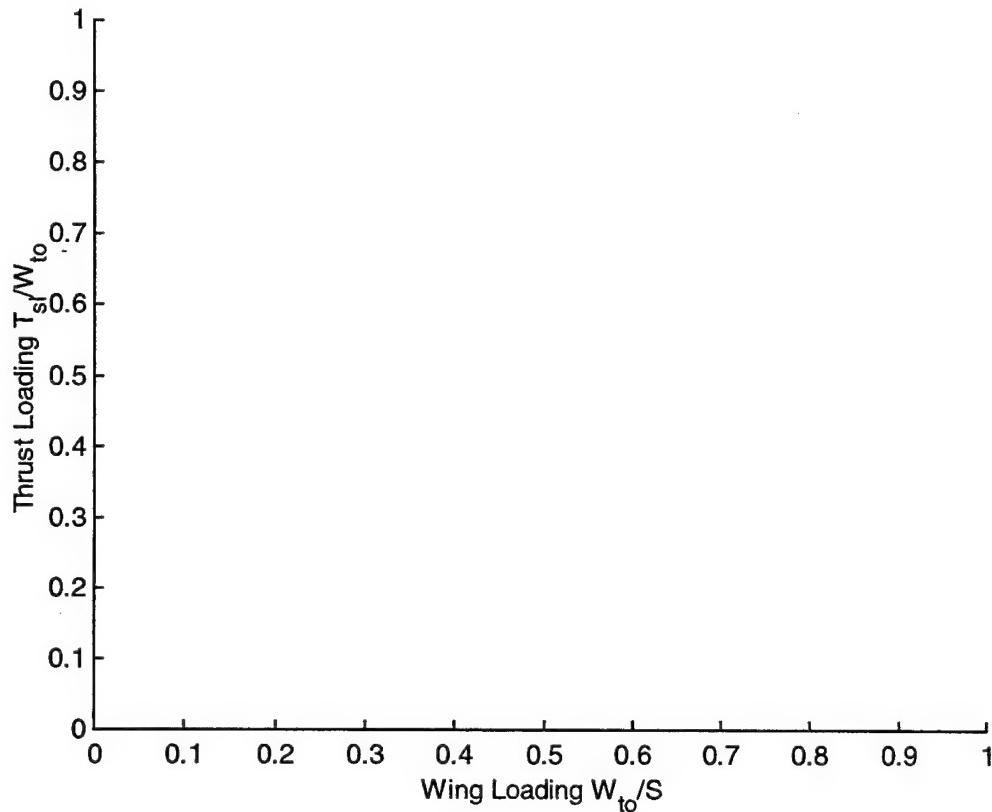
In order to formulate an expression in terms of our system parameters, the initial energy equation, (2.1), may be rewritten as

$$[T - (D + R)]V = W \frac{dh}{dt} + \frac{W}{g} \left( \frac{d}{dt} \left( \frac{V^2}{2} \right) \right). \quad (2.2)$$

This equation can be further modified with a few assumptions, which introduce the proposed system parameters, ( $W_{to}/S$ ) and ( $T_{sl}/ W_{to}$ ). The major assumptions concern the modeling or mathematical representation of the thrust, weight and drag.

### **1. Weight**

The weight of a configuration during any flight segment of a proposed mission may be expressed as a fraction of the takeoff gross weight. As the configuration proceeds



**Figure 1. Constraint Diagram Axes**

from one point in a mission to another, the fuel burn is usually considered to be the major contribution to weight change. For example, during the takeoff portion of the mission, the weight used in the expression (2.2) is the gross configuration weight. For landing, the loss of payload and fuel is considered. As the configuration proceeds through its mission,

the weight of the configuration decreases so that the weight fraction,  $\beta$ , decreases. Therefore, a model for computing the weight of the configuration at any time during a mission can be written as

$$W = \beta W_{to}. \quad (2.3)$$

## 2. Thrust

The modeling of the configuration's thrust variation with altitude is another concern. In general, an expression for the thrust of a jet-powered vehicle can be written as

$$T = \left( \frac{\dot{W}_{fuel} + \dot{W}_{air}}{g} \right) V_{jet} + (P_e - P_\infty) A. \quad (2.4)$$

This approach to representing the configuration's thrust is slightly cumbersome. A simpler model is desired. Hale proposes a more compact derivation of a model for the thrust variation with altitude. Hale suggests that the thrust of a jet-powered vehicle may be expressed as a function of altitude, free stream velocity and throttle control setting,

$$T = T(h_{altitude}, V, \Pi) \quad (2.5)$$

To further simplify this expression, Hale considers the condition where the control setting ( $\Pi$ ) is fixed. For use in this model, the control setting or throttle is considered to be at the full-open position. The thrust of a turbojet engine, for a given throttle setting, is directly proportional to the mass flow rate of the air through the engine (see equation 2.4). Consequently, as the density of the atmosphere decreases with an increase in altitude, the

available thrust decreases. The thrust at any given altitude can be expressed approximately in terms of its sea-level value. The ratio of available thrust at altitude to sea level thrust becomes [Ref. 11: p. 22-25]

$$\frac{T}{T_{sl}} = \left( \frac{\rho}{\rho_{sl}} \right)^x \equiv \sigma^x. \quad (2.6)$$

For flight in the stratosphere, a generally accepted value for the exponent is one. The thrust model can be written as [Ref. 10: p. 21]

$$T = \alpha T_{sl}, \quad (2.7)$$

where

$$\alpha = \sigma^1. \quad (2.8)$$

### 3. Drag Polar

The modeling of the airplane configuration drag is the next concern. An expression for the drag that allows an approximation of the total configuration drag is provided by Mattingly [Ref. 10: p. 20]. He chooses to use the expanded form of the drag polar representation

$$C_D = C_{D_0} + K_1 C_L^2 + K_2 C_L. \quad (2.9)$$

For first-order approximation, the constant ( $K_2$ ) is considered to be negligible since it concerns itself with the effects of system camber, which are small. The conventional drag polar expression takes the form

$$C_D = C_{D_0} + K C_L^2. \quad (2.10)$$

Additionally, the total drag of the configuration can be written as

$$D = qSC_D. \quad (2.11)$$

Utilizing the above models for weight and thrust and assuming the configuration remains clean, thus eliminating the second drag term ( $R$ ), a formulation of the general, first law constraint equation can be written as

$$\frac{T_{sl}}{W_{to}} = \frac{\beta}{\alpha} \left\{ \frac{D}{\beta W_{to}} + \frac{1}{V} \frac{d}{dt} \left( h + \frac{V^2}{2g} \right) \right\}. \quad (2.12)$$

Including the model for the drag polar, equation (2.12) can be written as

$$\frac{T_{sl}}{W_{to}} = \frac{\beta}{\alpha} \left\{ \frac{C_{D_0}}{\beta} \left( \frac{1}{\frac{W_{to}}{S}} \right) + K \frac{\beta}{q} \left( \frac{W_{to}}{S} \right) + \frac{1}{V} \frac{d}{dt} \left( h + \frac{V^2}{2g} \right) \right\}. \quad (2.13)$$

This equation can be modified to reflect many of the constraints that the operational requirements may impose on the configuration. For example, the constraint equation for horizontal acceleration (holding altitude constant;  $dh/dt = 0$ ) becomes

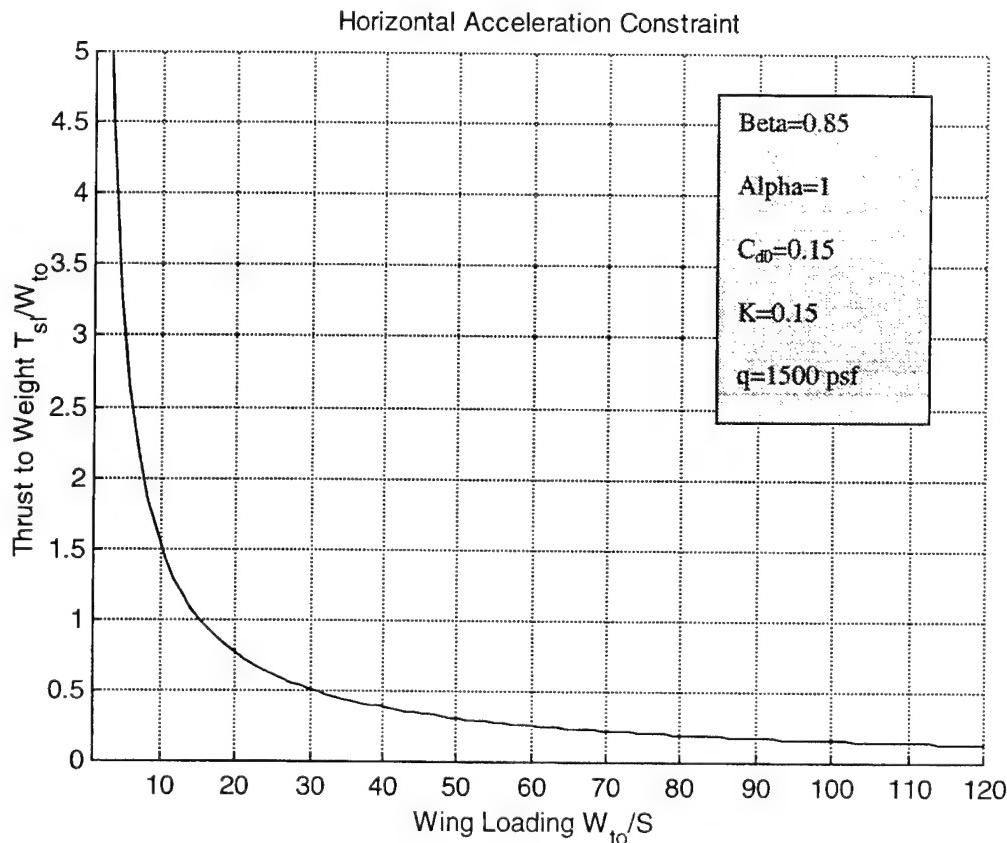
$$\frac{T_{sl}}{W_{to}} = \frac{\beta}{\alpha} \left\{ \frac{C_{D_0}}{\beta} \left( \frac{1}{\frac{W_{to}}{S}} \right) + K \frac{\beta}{q} \left( \frac{W_{to}}{S} \right) + \frac{1}{g} \left( \frac{dV}{dt} \right) \right\}. \quad (2.14)$$

An example of this formulation for an acceleration requirement is shown in Figure 2. Since this is an illustration, the vehicle type is not important, in and of itself, as the Figure is presented only to show the graphical representation of an acceleration requirement. From a mission profile and a given configuration, values for ( $\beta$ ,  $\alpha$ ,  $C_{D0}$ ,  $K$ ) as well as the dynamic pressure ( $q$ ) can be obtained. The values used for this illustrative

example are annotated in the figure. Substituting these values (which remain constant during the flight maneuver) into equation (2.14) yields a simple expression where the thrust loading is a function of only the wing loading.

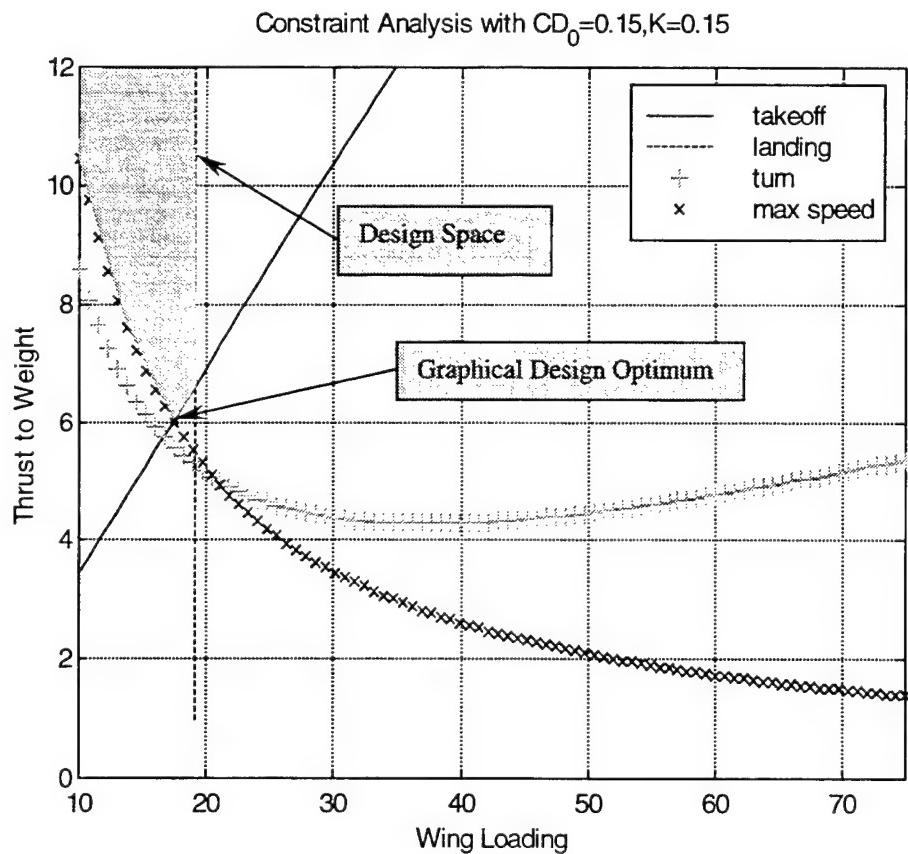
$$\frac{T_{sl}}{W_{to}} = f\left(\frac{W_{to}}{S}\right) \quad (2.15)$$

In Figure 2, the horizontal acceleration requirement is satisfied by the region above and to the right of the curve. Thus, the solution space for the horizontal acceleration requirement is any set of coordinates above and to the right of the acceleration constraint boundary.



**Figure 2. Horizontal Acceleration Constraint**

When the general, first law constraint expression is modified for each of the necessary operational requirements for the configuration and each of these is plotted on the same diagram, a constraint diagram results. A sample constraint diagram is shown in Figure 3. Constraint lines for takeoff, landing, sustained turn, and maximum speed requirements are depicted. Constraints that are not shown include, but are not limited to, instantaneous turn rate, deceleration and vertical climb rate. The air vehicle used for this example is a notional configuration for a light combat, forward air control vehicle, which was analyzed during coursework at NPS. The design solution space is shaded.



**Figure 3. Sample Constraint Diagram**

The constraint diagram allows for a two-dimensional, graphical representation of the operational requirements placed on a configuration. The resulting design or solution space (shaded in Figure 3) represents the set of parameters, wing loading ( $W_{to}/S$ ) and thrust-to-weight ratio ( $T_{sl}/W_{to}$ ) that simultaneously satisfy all of the mission requirements. The graphical optimum, as shown in Figure 3, represents the “best” system solution for the requirement set for our design.

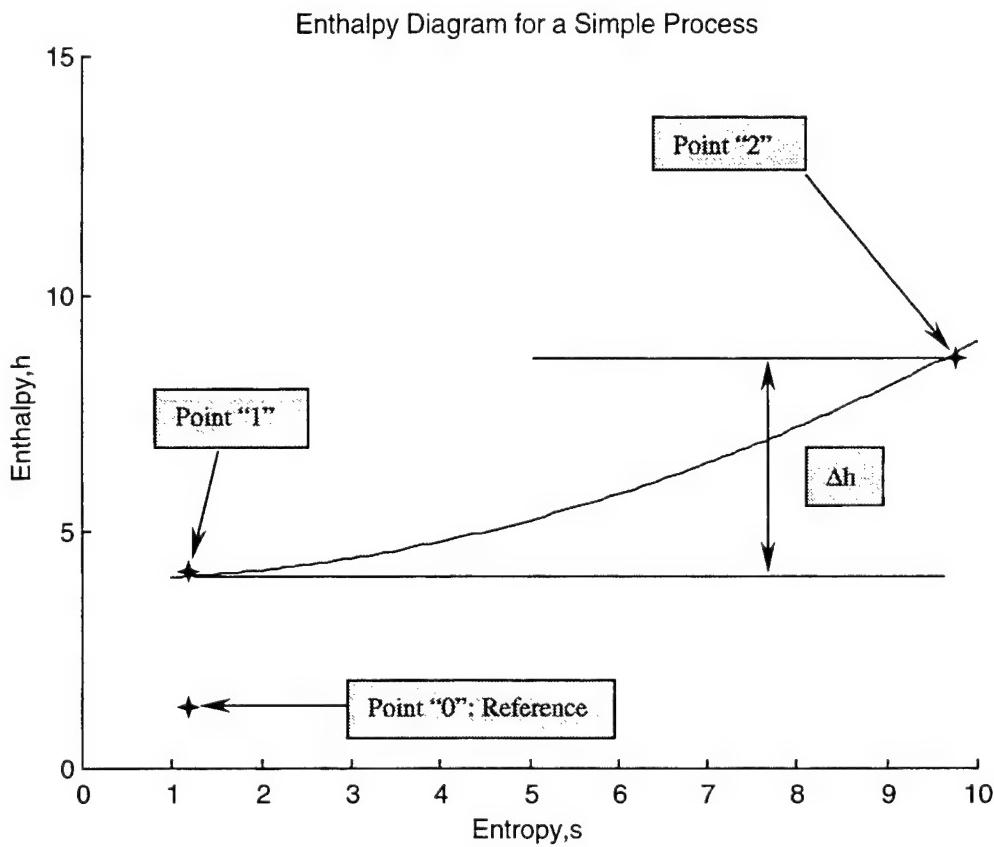
## B. AVAILABILITY AND EXERGY

The First Law of Thermodynamics states that the heat entering a system equals the work done by the system for a closed process. The First Law provides no quantitative information about the distribution of energy within the system or the quality of the work performed by the system. From our previous discussion of the general, First Law constraint expression it can be seen that no information about the quality or usefulness of the configuration is provided by the two-dimensional representation. Inclusion of the Second Law into the conceptual design process may provide insight into the distribution and quality of the energy and work within the aeronautical system.

The Second Law of Thermodynamics accounts for the entropy increase within a system for any given process. The Second Law also provides information about the distribution of energy within a system and the quality of the work performed by the system. The thermodynamic variable exergy (*Ex*), or availability (*A*), can be used to describe the quality and distribution of energy within a system. The thermodynamic variable associated with entropy gain due to a process can be defined as  $\sigma$  (not to be

confused with the density ratio). Additional information about the entropy gain due to a process is helpful in identifying the “entropy producers” within the system undergoing a process.

To initiate development of the concept of exergy, consider the simple process shown in Figure 4. Point “0” is used to define a reference state point and state points “1” and “2” define the start and end points of our sample process.



**Figure 4. Sample Process Enthalpy Diagram**

The following simple derivation, following Ahern’s work, is used to develop a definition of exergy [Ref. 12: p. 47].

$$\text{Available Work} = \text{Exergy} = (h - h_0) - T_0(s - s_0). \quad (2.16)$$

Expanding this formulation to represent the available work between the two points on our diagram (Figure 4) yields

$$\text{Available Work}_{(1,2)} = (h_2 - h_1) - T_0(s_2 - s_1). \quad (2.17)$$

Ahern's analysis of this equation reveals that the overall conditions of the environment have a much more significant effect on system efficiency and are prominent in the lost-work analysis [Ref. 12: p. 46]. Thus, the loss of available work becomes

$$\text{Loss of Available Work} = T_0(s_2 - s_1). \quad (2.18)$$

The loss of available work or irreversibility ( $I$ ) of a process can be written as

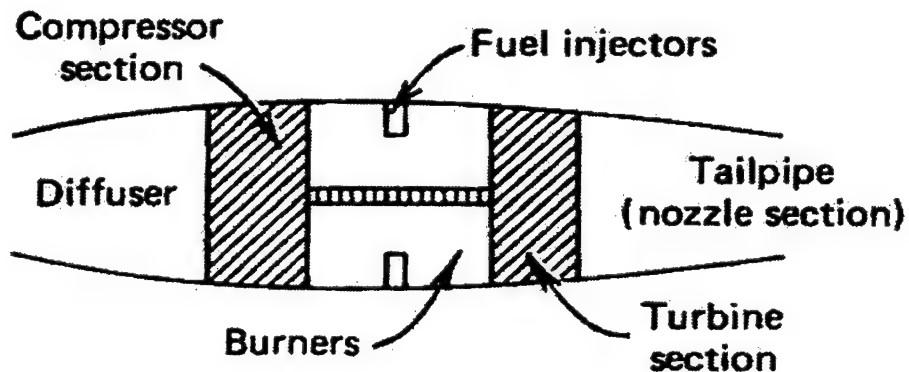
$$I = T_0\sigma; \{\sigma \text{ here is the entropy gain, not density ratio}\}. \quad (2.19)$$

This thermodynamic quantity is useful for identifying the subsystems or components of a subsystem that contribute to the overall irreversibility of a system. Such subsystems and components can be referred to as "entropy producers".

### C. AVAILABILITY ANALYSIS

The use of an availability analysis provides significant insight into the quality of inefficient energy use. [Ref. 12: p. 48] The combined use of the First and Second laws of thermodynamics is required to determine the quality, quantity and distribution of the energy within a system. Both laws are necessary, as they are complimentary. The result of an exergy, or availability analysis is a qualitative and quantitative accounting of the energy in a system. [Ref. 13: p. 7] For example, a thermodynamic analysis of a turbojet,

based on Moran's work, suggests that the available energy to do work comes only from the fuel. The chemical availability of the fuel is calculated using the Gibb's free energy function in order to determine the overall amount of energy available to the turbojet. An availability analysis permits the computation of the distribution or redistribution of this available energy. The quality of the work performed by the turbojet can also be determined in the availability analysis. The quality of work, or irreversibility, of each of the four main engine components (compressor, burner, turbine and exhaust nozzle) is then computed and the total irreversibility of the turbomachine is presented to show which components are the major "entropy-producers". Figure 5 is a general schematic of a turbojet engine. It is provided to show a standard turbine engine and to illustrate the four main components referenced in the availability analysis.



**Figure 5. General Engine Schematic [Ref. 11]**

The information in Table 1 illustrates the steady-state operating data for a turbine engine. For all sections the velocity of the fluid is given relative to the surrounding air.

**Table 1: Steady-State Operating Data**

Section	Fluid	Temp (°K)	Pressure (bars)	Velocity (m/s)	Flow Rate (kg/s)
Free Stream	Air	288	~1	0	1
Diffuser	Air	308	1.2	—	1
Compressor	Air	550	7.5	—	1
—	Fuel	320	11	—	0.025
Combustor	Mixture	1695	8	—	1.025
Turbine	Mixture	1447	4.5	—	1.025
Exit Nozzle	Mixture	988	~1	755	1.025

The information in Table 2 summarizes the available energy at each of these sections and the calculated irreversibility produced by each component of the turbomachine.

**Table 2: Summary of the Availability for a Turbine Engine**

	KJ/s	Percent of Fuel
		Availability
Availability in (at Inlet)	0	—
Availability in (with Fuel)	1350	—
Availability out, Nozzle Exhaust	<b>720</b>	<b>53</b>
<b>Irreversibilities</b>		
Diffuser	0.39	0.03
Compressor	20	1.53
Combustor	392	29.05
Turbine	5.58	0.40
Nozzle, Jet Pipe	4.4	0.35
<b>Total Irreversibilities of Components</b>	<b>422.37</b>	<b>31.36</b>

Perhaps the most important conclusion from this analysis is that almost 53% of the fuel's availability is carried out in the exhaust stream. Each component of the engine contributes to the overall irreversibility, with the combustor contribution (29%) being the most significant. For the overall turbojet analysis, approximately 84% (53% + 31%) of

the fuel availability is either destroyed by irreversibilities or carried out in the exhaust [Ref. 13: p. 184-188].

This variation on Moran's work highlights the desirable effects of availability analyses. Thus, insights can be gained into the efficiency and effectiveness of a system during the conceptual design phase when cost effective design decisions can be made.



*“[Availability analyses are] useful in directing the attention of process engineers, research engineers and technical managers to those aspects which offer the greatest opportunities for improvement.”*

**R. Gatts, R. Massey and J. Robertson**



### **III. SECOND LAW CONCEPTS IN DESIGN**

The goal of this thesis is to provide one method for incorporating both the First and Second Laws of Thermodynamics into the conceptual design process. The previous chapters have narrated the historical motivation for this idea and provided reference material supporting the idea of and necessity for design related Second Law analyses. The basic concepts of design space and the primary variables, or parameters used in a Second Law analysis of a configuration in the conceptual design phase have been developed. In this chapter, Second Law variables are incorporated into the First Law constraint expression and two important, First Law concepts are reconstructed using a redefined general constraint expression. An attempt to relate the original, First Law expressions of the constraint equation from Mattingly (equation 2.13) and Donaldson and Jones' generic, hypersonic, vehicle weight fraction will be shown. These concepts will be used to suggest partial verification of the proposed Second Law expression.

#### **A. CONSTRAINT EQUATIONS**

An alternative formulation of the general, First Law constraint expression recast in terms of variables related to the Second Law is desired so that access to the design space may be gained through terms that are meaningful in a Second Law analysis. The development of a Second Law expression for the First Law constraint equation presented in the previous chapter begins with the work of Czysz and Murthy in their 1989 paper.

Their explanation of the energy balance for a turbojet powered aircraft initiates the derivation of a Second Law constraint expression,

$$(KE + PE) + (FrictionHeat/Drag) = (CombustionHeat) + (Heat_{recoverable}). \quad (3.1)$$

Equation (3.1) shows the decomposition of the energy within a typical aircraft system. For ease of expression, we have termed the heat developed by friction and drag as “AeroHeat<sub>lost</sub>” and the recoverable heat term as “AeroHeat<sub>recoverable</sub>”.

Since the constraint expression, in either the First or Second Law variation, is a rate equation, consider the time derivative of equation (3.1).

$$\frac{d}{dt}(KE + PE) + \frac{d}{dt}(AeroHeat_{lost}) = \frac{d}{dt}(CombustionHeat) + \frac{d}{dt}(AeroHeat_{recoverable}) \quad (3.2)$$

Looking at these terms individually yields

$$\frac{d}{dt}(KE + PE) = \frac{d}{dt}\left(h + \frac{W}{g} \frac{V^2}{2}\right) = \frac{d}{dt}\left(\frac{W}{g} h + \frac{d}{dt}\left(\frac{W}{g} \left(\frac{V^2}{2}\right)\right)\right), \quad (3.2a)$$

$$\frac{d}{dt}(AeroHeat_{lost}) = \frac{d}{dt}\left\{(DVdt) + \dot{m}C_p(T_0 - T_\infty)\right\}, \quad (3.2b)$$

$$\frac{d}{dt}(CombustionHeat) = \frac{d}{dt}(f * (\Delta H_c)) = \frac{d}{dt}\left(\frac{W_f}{W_{tot}} * (\Delta H_c)\right) = \frac{\dot{W}_f}{W_{tot}} * (\Delta H_c), \quad (3.2c)$$

$$\frac{d}{dt}(AeroHeat_{recoverable}) = \frac{d}{dt}(KE + PE + AeroHeat_{lost} - CombustionHeat). \quad (3.2d)$$

Equation (3.2d) may first appear to be redundant but it provides a form for the Second Law expression that is analogous to Mattingly's general, First Law, constraint equation. To fully define an expression for the derivative of the “AeroHeat<sub>recoverable</sub>”, the quantities

in equation (3.2d) require an explicit form. After substitution of (3.2a-c) into (3.2d), an explicit form of (3.2d) can be rewritten as,

$$\begin{aligned} \frac{d}{dt} (AeroHeat_{recoverable}) = & \left\{ \frac{\dot{W}V^2}{2g} + \frac{W}{g}V \frac{dV}{dt} + \dot{W}h + W \frac{dh}{dt} \right\} + \\ & \left\{ DV + \frac{\dot{W}}{g} (C_p (T_0 - T_\infty)) \right\} - \left\{ \frac{\dot{W}_{fuel}}{W_{air}} (\Delta H_c) - \frac{W_{fuel} \dot{W}_{air}}{W_{air}^2} (\Delta H_c) \right\} \end{aligned} \quad (3.3)$$

This definition of the time derivative of the recoverable aerodynamic heat expresses the system energy in terms of aircraft performance variables. Additionally, equation (3.3) shows the decomposition of the energy of the system and the total quantity of energy available to perform work (assuming, as was done in the availability analysis of II-C, that all availability to perform work comes from the fuel). Examination of the quantities in (3.3) shows that the first term in braces contains the quantities concerned with the change in the kinematics of the system. The second term in braces describes the heat lost to the irreversibilities of aerodynamic friction and the total configuration drag. The third term in braces describes the energy available to the system through the combustion of the fuel. Equation (3.3) allows insight into the availability and irreversibility of our design.

Further simplification of equation (3.3) is necessary to produce an expression of the First Law, general constraint equation in terms of variables related to the Second Law. Initial simplification of (3.3) can be accomplished by using the weight and drag models provided in Chapter II (sections A.1, A.2). The thrust model will be used with one minor change. The exponent's value in equation (2.8) is changed to seven-tenths (*i.e.*,  $x = 0.7$ ).

This value better represents the “density lapse rate” within the troposphere, which is the area chosen for the analysis herein.

One new term enters the expression via simplification, which needs a detailed description. This new quantity is the effective area.

### 1. Effective Area Model

The effective area is a better representation of the volume of fluid affected by our system than the projected area ( $A_{proj}$ ) [Ref. 14]. The effective area ( $A_{eff}$ ) enters the equation through the representation of the time derivative of the weight of the air. The

$$\dot{W}_{air} = \rho_{sl} \sigma^{0.7} V A_{proj} \left( \frac{A_{eff}}{A_{proj}} \right) \quad (3.4)$$

The effective area is the planar area of the volume captured by the configuration. We have chosen to represent this based on the projected area of the configuration. Thus, the model for the effective area is

$$A_{effective} = 2A_{proj}. \quad (3.5)$$

The expression for the First Law-based, general constraint equation in terms of variables related to the Second Law can now be written in a simplified form. With substitution of the models for thrust, weight and effective area into equation (3.3), the final expression becomes

$$\frac{T_{sl}}{W_{to}} = \left( \frac{\beta \dot{m}_{air} g}{2q\sigma^{0.7} A_{eff}} \right) \frac{d}{dt} \left( h + \frac{V^2}{2g} \right) + \frac{\dot{m}_{air} C_D}{2\sigma^{0.7} W_{to}/S}. \quad (3.6)$$

Now that this representation of the general constraint expression is in terms of variables related to the Second Law, verification and validation of this representation can be performed.

## B. CONSTRAINT ANALYSIS - REVISITED

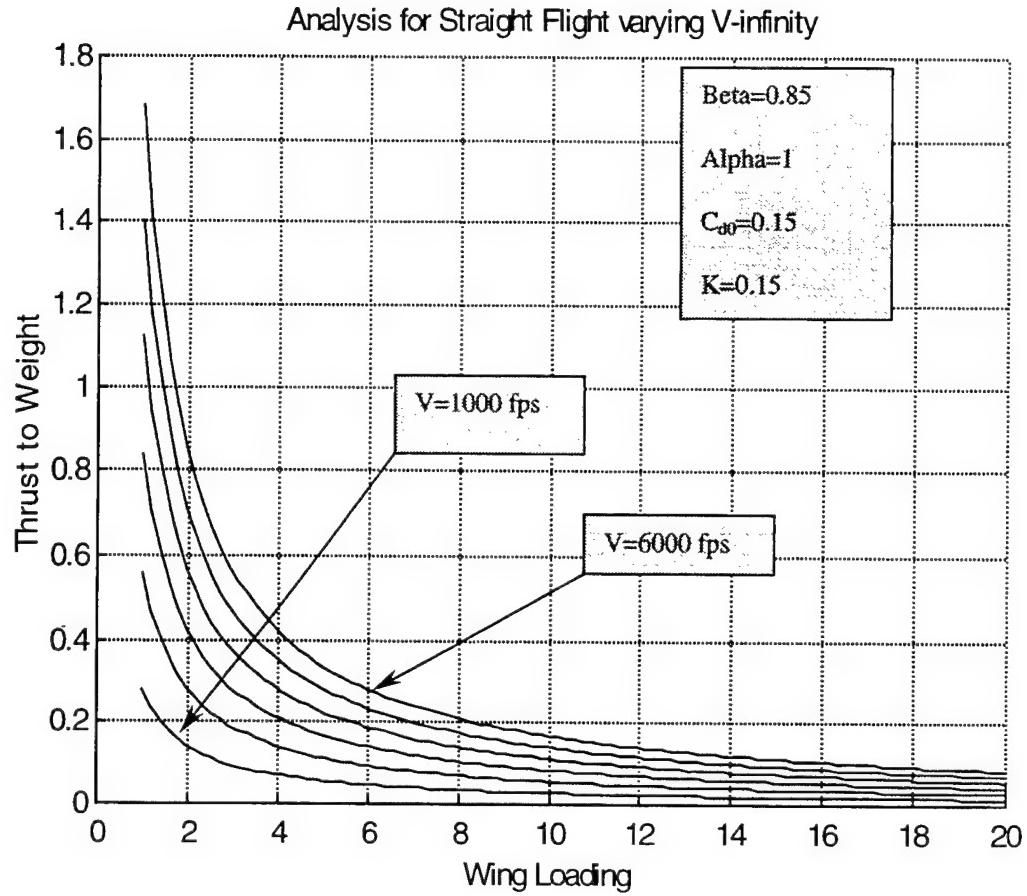
In order to verify, at least partially, the representation of the general constraint expression, specific constraints (e.g., level unaccelerated flight and horizontal acceleration) were mathematically constructed. This construction was performed to compare the Second Law, general constraint expression against the First Law constraint equation under similar conditions. These specific constraint constructions were then compared graphically to the First Law counterparts for further validation. For additional verification of similarity to the First Law, some of the Second Law variables in the specific constraint expressions were modified to determine whether or not the new representation mimicked the First Law constraint behavior.

Two specific, aircraft system requirements that might serve as operational constraints were considered during this verification of the Second Law representation. The first requirement constructed was the level, unaccelerated flight condition. This condition was chosen because it represents the simplest requirement. The second requirement considered in the verification of the Second Law expression was the condition for horizontal acceleration. This was chosen since it also represents a simple constraint while introducing a greater number of Second Law variables, without introducing too many additional First Law variables.

The representation of the level, unaccelerated flight condition can be obtained by simplifying equation (3.6). The simplification can be performed in two parts. The first simplification is that level flight implies that altitude remains constant and therefore, the time derivative of the altitude is zero. The second simplification is that the condition for “unaccelerated” implies that the time derivative of the velocity is also zero. Using these conditions to simplify (3.6), one obtains,

$$\frac{T_{sl}}{W_{to}} = \frac{\dot{m}_{air} C_D}{2\sigma^{0.7}} \left( \frac{1}{\frac{W_{to}}{S}} \right). \quad (3.7)$$

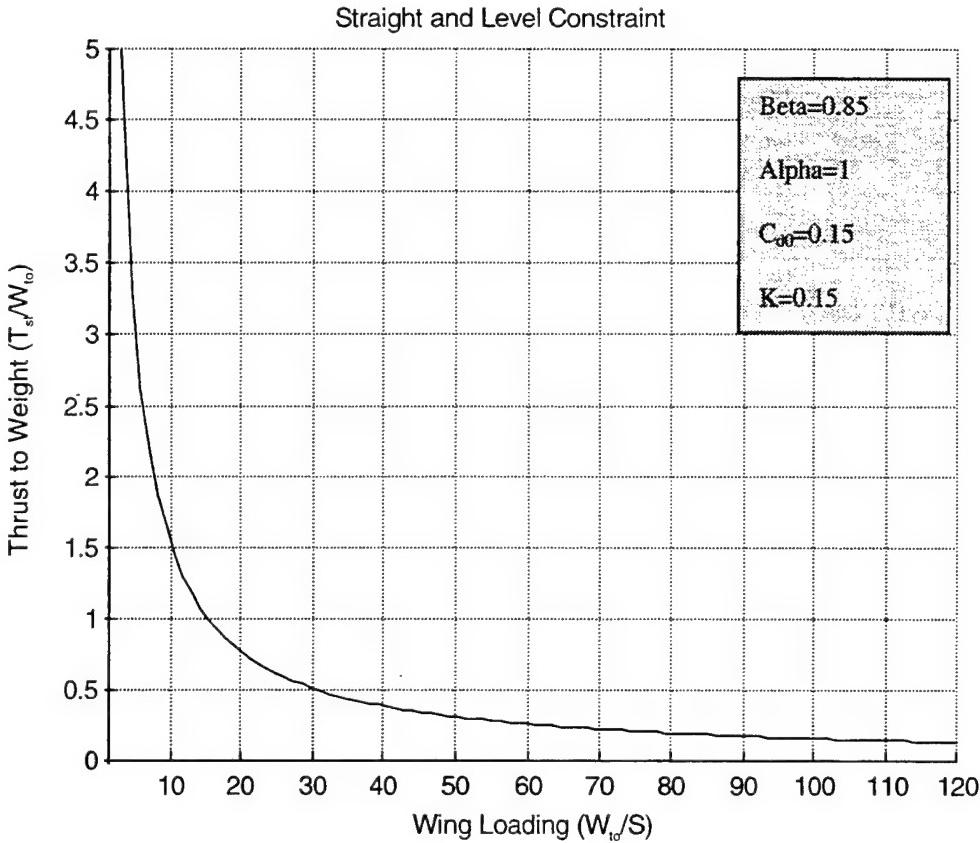
Figure 6 was obtained by selecting a range of wing loadings and choosing values for the configuration’s velocity (Mach 1), altitude (30,000 feet), projected area ( $10 \text{ ft}^2$ ), and drag coefficients ( $C_{D0} = 0.15$ ,  $K = 0.15$ ,  $C_L = 1$ ). Figure 6 shows that the constraint line has the predicted shape from the First Law representation of the level, unaccelerated requirement (Figure 7) under similar conditions. To investigate whether or not this specific constraint behaves in similar fashion to the First Law expression, the freestream velocity was increased from the intial value of 1000 fps in increments 1000 fps for each of the subsequent graphings of the specific, Second Law constraint for level, unaccelerated conditions. The movement of the constraint line, as the freestream velocity is increased, is “up and to the right”, which seems to replicate the behavior of the First Law expression under the same conditions.



**Figure 6. Second Law Level Flight Constraint (vary V-infinity)**

The second requirement constructed was the horizontal acceleration condition. Equation (3.6) was modified to include the proper simplifications, specifically that the time derivative of the altitude is zero ( $dh/dt = 0$ ). Including this condition in (3.6), the horizontal acceleration constraint expression simplifies to,

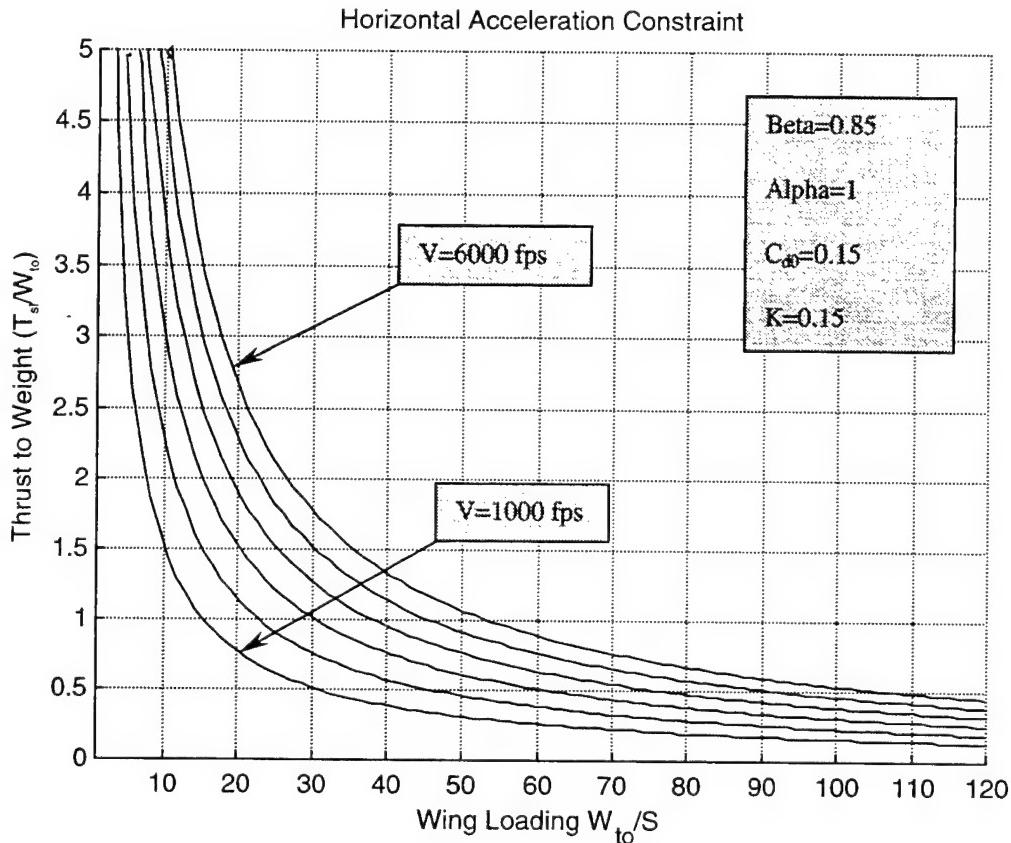
$$\frac{T_{sl}}{W_{lo}} = \frac{\beta \dot{m}_{air}}{2q\sigma^{0.7} A_{eff}} \left( V \frac{dV}{dt} \right) + \frac{\dot{m}_{air} C_D}{2\sigma^{0.7}} \left( \frac{1}{\frac{W_{lo}}{S}} \right). \quad (3.8)$$



**Figure 7. First Law Level Flight Constraint**

This constraint is plotted in Figure 8 using the same values for the same variables in Figure 6. When compared with Figure 2, the curve in Figure 8 replicates the behavior of the First Law expression under the horizontal acceleration conditions. To further investigate the behavior of the Second Law expression, the freestream velocity was increased (as in Figure 6) for subsequent evaluations of the Second Law expression.

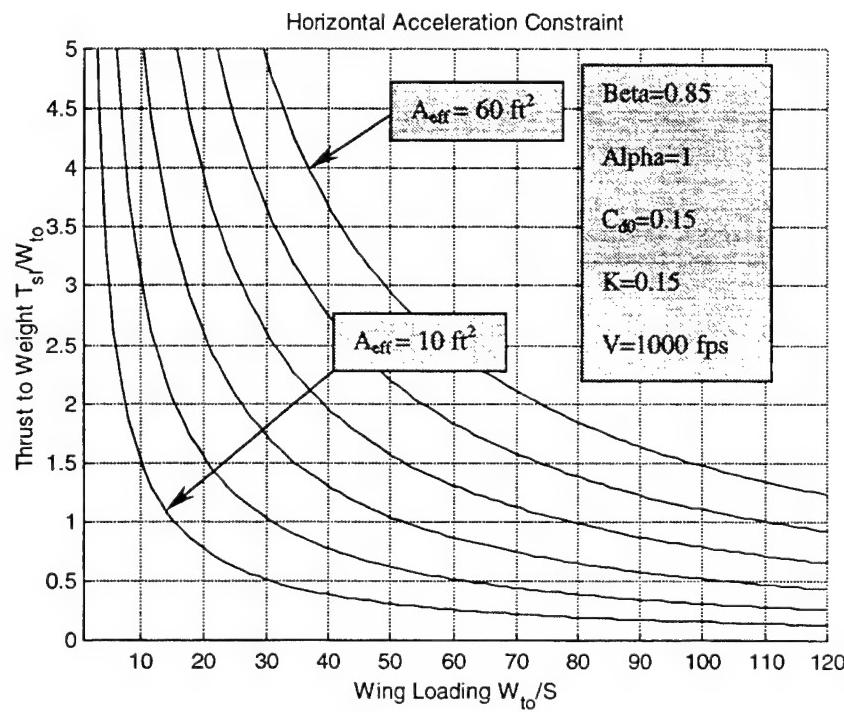
Figure 8 shows that the horizontal acceleration constraint does move “up and to the right” with an increase in freestream velocity, which behaves like the First Law constraint equation under similar conditions. Thus, the First Law constraint equation can be derived from the Second Law expression.



**Figure 8: Horizontal Acceleration Constraint Plot (varying  $V_{\infty}$ )**

Since the horizontal acceleration requirement was selected because it introduced additional variables related to the Second Law, the effective area was modified while holding the freestream velocity constant (1000 fps). The effective area started with a value of one square foot ( $1 \text{ ft}^2$ ) and was increased by ten ( $10 \text{ ft}^2$ ) for each subsequent

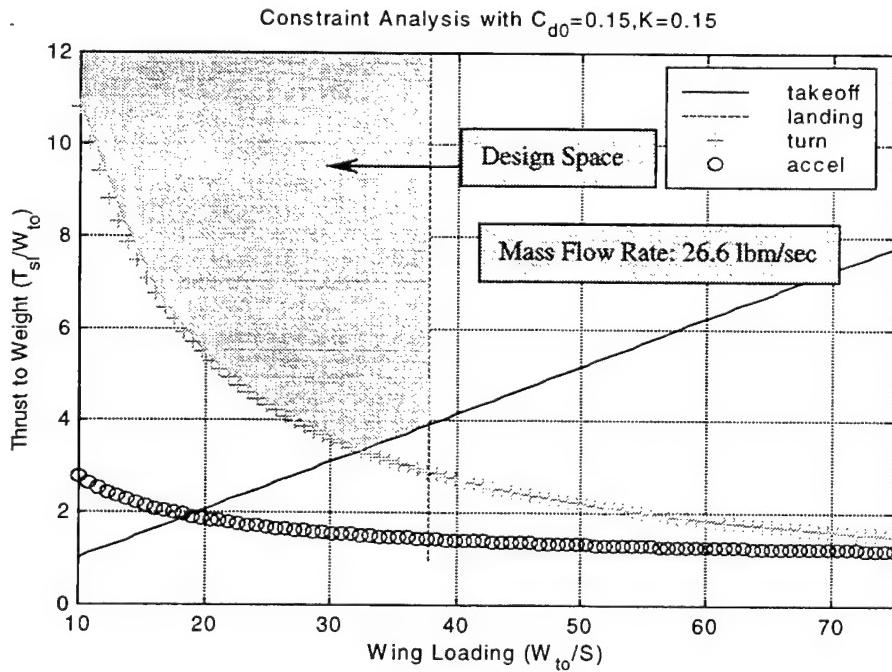
iteration. Figure 9 shows the clustering of the constraint lines at the extreme values of wing loading, while the apexes of the curves spread. This trend is proper since an increase in effective area, according to our model for effective area, is an increase in the projected area of the configuration. Thus, an increase in effective area equates to an increase in overall size of the configuration and an increase in configuration size necessarily increases the configuration drag and subsequently requires a higher thrust loading at any given freestream velocity.



**Figure 9. Horizontal Acceleration Constraint Plot (varying  $A_{\text{effective}}$ )**

As a test of the Second Law constraint expression a constraint diagram was constructed. The operational requirements considered for the conceptual configuration were level flight, horizontal acceleration, takeoff and landing distances, level turn and a

maximum speed requirement. The configuration was required to cruise at Mach 6, to accelerate to Mach 8, and to maintain a level, 1-g turn at 30,000 feet. The general expression (3.6) was modified according to the conditions of each requirement. Each of these requirements was plotted on the same diagram in order to construct the constraint diagram. Figure 10 is the constraint diagram for our conceptual idealization of a hypersonic vehicle and is intended to illustrate the viability of the Second Law representation. The design space is shaded. Figure 10 shows that the main constraints defining the solution space are the turn and takeoff requirements.



**Figure 10. Constraint Diagram using Second Law Expression**

### C. FUEL WEIGHT FRACTION

The first partial verification of the Second Law representation, equation (3.3), was to develop an expression using Second Law variables that was analogous to the First Law expression. A second verification of the Second Law representation (3.3) can be made by considering the Donaldson and Jones' hypersonic airbreathing vehicle analysis. The recreation of their specific weight fraction is necessary since the weight fraction is one of the few quantities that allow design freedom during the conceptual design phase. [Ref. 6: p. 34] The Donaldson and Jones weight fraction is

$$\beta = \frac{W_{final}}{W_{initial}} = \left\{ 1 - \left\{ 1 - \frac{V_0^2/2 + gh_0}{2\eta_p Q(T/D-1)} \right\} \div \right. \\ \left. \left\{ 1 + \frac{V_0^2/2 + gh_0}{\eta_p Q} \left[ 1 + \frac{1}{2((T/D-1))} \right] \right\} \right\}. \quad (3.9)$$

Our representation of this weight fraction begins with equation (3.2d),

$$\frac{d}{dt}(AeroHeat_{recoverable}) = \frac{d}{dt}(KE + PE + AeroHeat_{lost} - CombustionHeat). \quad (3.2d)$$

The complete derivation of the weight fraction from (3.1) appears in the Appendix and is based on the original derivation in Czysz's unpublished notes. [Ref. 14] The final representation of the weight fraction is,

$$\beta = \frac{1 + E_f \left[ 1 + \frac{(1 - \chi_i)^2}{2(T/D - 1)} \right]}{1 + E_i - E_f \left[ \frac{(1 - \chi_i^2/2)}{2(T/D - 1)} \right]}. \quad (3.10)$$

The following definitions are given for ease of the reader:

$$e \equiv gh + \frac{V^2}{2},$$

$$E \equiv \frac{e}{\eta_p J Q g},$$

$$\chi \equiv \frac{e}{e_f},$$

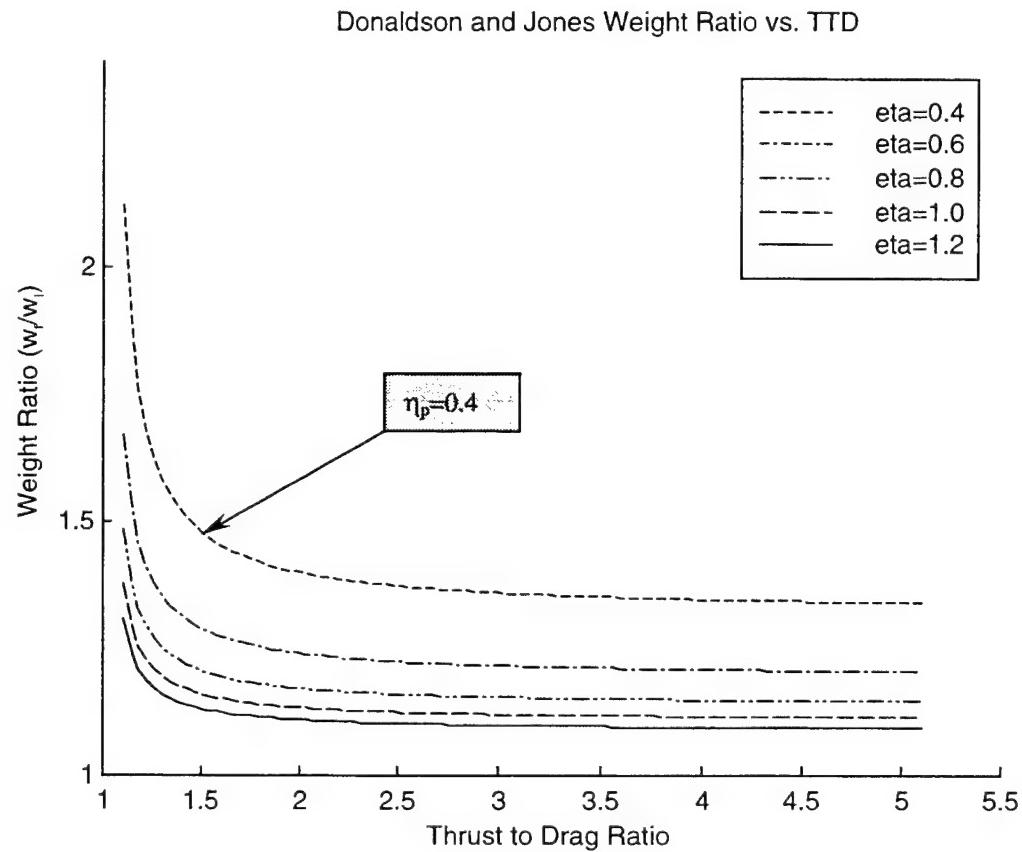
$$\text{then } \chi_i = \frac{e_i}{e_f} = \frac{1}{W_r}.$$
(10 a-d)

In order to reproduce Donaldson and Jones's weight fraction, ( $E_i$ ) and ( $\chi_i$ ) need to be zero. With these conditions fulfilled, the weight fraction in equation (3.10) may be plotted for comparison with Donaldson and Jones' expression (3.9). The value of the propulsive efficiency ( $\eta_p$ ) begins at 0.4 and is incremented in steps of 0.2 in Figure 11.

Figure 11 shows our reconstruction of Donaldson and Jones's weight fraction curves.

Comparison of our curves to Donaldson and Jones's figure shows similar curve structure and dependence on propulsive efficiency ( $\eta_p$ ). In both our recreation and the original, the curves are steep at the outset and flatten considerably. The dependence on propulsive efficiency is seen in the curve moving vertically down with an increase in efficiency, which implies that a lower weight fraction is gained through propulsive efficiency gains.

With the aforementioned examples, it is suggested that the Second Law representation, equation (3.3), can be used to reproduce the First Law constraint diagram for several different requirements and it can also reproduce the critical parameter of the weight fraction.



**Figure 11: Donaldson/Jones Weight Fraction**

## IV. SECOND LAW ANALYSIS

This study has developed an approach for one method of incorporating the First and Second Laws into the conceptual design phase. This method expresses the First Law constraint equation in terms of variables related to the Second Law. The use of these variables for Second Law analysis is the next step in incorporating the First and Second Laws into the conceptual design phase. One possible application for these variables is to develop a method for evaluating the efficiency of a configuration. Preliminary ventures in this area have led to one possible definition for the overall thermal efficiency of a configuration.

The initial concept for a definition of thermal efficiency comes from conventional Second Law analyses. These analyses have defined the efficiencies of classical thermodynamic cycles (e.g., Carnot, Brayton, etc.). A typical definition for efficiency may be [Ref. 12: p. 32]

$$\eta_{engine} = \frac{\left( \frac{\text{Work Out}}{\text{Energy In}} \right)}{\eta_{ideal_{Carnot}}}. \quad (4.1)$$

This definition is convenient for First Law analysis, but contains no information about the irreversibility of the process or the loss of energy. A proposed definition for efficiency that includes energy losses is [Ref. 12: p. 103]

$$\eta_{exergy} = \frac{\text{Maximum Exergy} - \sum \text{Exergy Loss}}{\text{Maximum Exergy}}, \quad (4.2)$$

which, for a closed cycle, is the same as the thermal efficiency [Ref. 12: p. 104]

$$\eta_{\text{thermal}} = \frac{\text{Useful Work}}{\text{Heat Input}} = \eta_{\text{exergy,closed}}. \quad (4.3)$$

Equation (4.3) contains terms that have possible interpretations within the conceptual design process. From the concepts of exergy and irreversibility, it may be possible to define (4.3) in terms of variables related to the Second Law. Additionally, these variables may be associated with the Second Law, general expression (3.3) and further related to the Second Law, constraint equation. If these relationships can be shown then a method for fully incorporating a Second Law analysis into the conceptual design process may be developed. To initiate the redefinition of (4.3) into the terminology of this thesis, one may return to equations (2.16), (2.17) and (2.18). Using the latter two, the useful work can be expressed as

$$\text{Useful Work} = \text{Available Work} - \sum \text{Irreversibilities}. \quad (4.4)$$

The irreversibilities for the configuration may be expressed as

$$\sum \text{Irreversibilities} = \sum_{\text{configuration}} T_0 \sigma_{\text{systems}}, \quad (4.5)$$

where the entropy gains ( $\sigma$ ) may be expressed as

$$\sum_{\text{configuration}} \sigma_{\text{system}} \approx \sigma_{\text{aerodynamics}} + \sigma_{\text{propulsion}} + \sigma_{\text{controls}} + (\dots) \quad (4.6)$$

There are many subsystems in a modern aircraft that potentially contribute entropy gains. Equation (4.6) is not intended to be necessarily inclusive. If this formulation of the irreversibilities is substituted into (4.4) and then further substituted into (4.3), one obtains

$$\eta_{exergy,closed} = \frac{\text{Available Work} - (\sigma_{aerodynamics} + \sigma_{propulsion} + (...))}{\text{Heat Input}}. \quad (4.7)$$

Dividing the heat input into the individual terms and then applying the definition of exergy (2.16) and the definition of exergy efficiency (4.2) gives

$$\eta_{exergy,closed} = \frac{\text{Available Work}}{\text{Heat Input}} - (\eta_{exergy,aerodynamics} + \eta_{exergy,propulsion} + [...]). \quad (4.8)$$

If the configuration could be thought of as a closed system, then equation (4.8) generates a definition for the overall efficiency of a configuration in terms of variables related to the Second Law.

$$\eta_{configuration} = \frac{\text{Available Work}}{\text{Heat Input}} - \sum \eta_{systems} \quad (4.9)$$

This definition appears to have beneficial properties for evaluating the efficiency of a configuration. This equation has not been evaluated for any configurations and requires further exploration, development and validation.



## V. CONCLUSIONS

The goal of this thesis is to provide one method for incorporating the First and Second Laws into the conceptual design phase. One method has been presented and partially validated. A general Second Law expression for the decomposition of the energy in a configuration has been derived. From this general expression, the First Law constraint equation was partially reproduced. The construction of the First Law constraint equation in terms of variables related to the Second Law provides access to the design space with follow-on applications towards a Second Law analysis. Additionally, the high-speed vehicle fuel weight fraction of Donaldson and Jones, a critical design parameter, has been partially reconstructed from the general Second Law expression. Both of these reconstructions provided partial validation of the general, Second Law expression through mathematical and graphical comparisons to their First Law counterparts.

Finally, a possible definition of the overall configuration efficiency in terms of variables related to the Second Law was presented. This proposed definition of overall configuration efficiency demonstrates that a high level of system integration for evolving vehicles is necessary. From the definition it can be seen that reducing the entropy gain of individual subsystems may increase the overall efficiency of the configuration. Altering the characteristics of one subsystem may have negative impacts on other subsystems and their efficiencies. Through system integration a possibly higher overall efficiency may be

obtained by considering or preventing the effects of subsystem changes before the design process proceeds to a stage where change is either impractical or cost ineffective.

Further work in the area of this thesis is necessary. The full validation of the general, Second Law expression needs completion. The proposed definition of overall configuration requires a rigorous mathematical construct and then needs to be tested. The testing procedure for this definition appears to be difficult since any existing definition of a configuration is not based on the Second Law of Thermodynamics.

## APPENDIX: DERIVATION OF FUEL WEIGHT FRACTION

The derivation of the expression for our weight fraction ( $\beta$ ) is based on the unpublished notes of Czysz. This appendix presents a simple description and outline of this derivation.

Beginning with the Donaldson and Jones integrated energy equation

$$\frac{d\left(gh + \frac{V^2}{2}\right)}{dt} = \frac{DV\left(\frac{T}{D} - 1\right)}{m} = \frac{(T-D)V}{m} \quad (\text{A.1})$$

We can express the time rate of change of the AeroHeat<sub>recoverable</sub> as

$$\frac{d(\text{AeroHeat}_{\text{recoverable}})}{dt} = DV = \left( \frac{d\left(gh + \frac{V^2}{2}\right)}{dt} \right) \left( \frac{m}{\left(\frac{T}{D} - 1\right)} \right) \quad (\text{A.2})$$

which may be simplified to

$$\frac{d(\text{AeroHeat}_{\text{recoverable}})}{d\left(gh + \frac{V^2}{2}\right)} = \frac{m}{\left(\frac{T}{D} - 1\right)} \quad (\text{A.2a})$$

Integrating this to get the aerodynamic heat recoverable

$$\text{AeroHeat}_{\text{recoverable}} = \int_i^f \frac{m}{\left(\frac{T}{D} - 1\right)} d\left(gh + \frac{V^2}{2}\right) \quad (\text{A.3})$$

The energy required to get to a desired velocity is comprised of the sum of the kinetic, potential and energy left in the wake as heat. The change in energy between two points may be written as

$$\begin{aligned}\Delta(KE + PE) &= \frac{W_f}{g} \left( gh_f + \frac{V_f^2}{2} \right) \left( 1 - \frac{W_r}{E_r} \right) \\ \beta &\equiv W_r \equiv \frac{W_i}{W_f} \\ e &\equiv gh + \frac{V^2}{2} \\ E &\equiv \frac{e}{\eta_p J Q g} \\ E_r &\equiv \frac{E_i}{E_f} = \frac{\left( gh_f + \frac{V_f^2}{2} \right)}{\left( gh_i + \frac{V_i^2}{2} \right)}\end{aligned}\tag{A.4}$$

Define

$$\begin{aligned}\chi &\equiv \frac{e}{e_f} \\ \text{then } \chi_i &= \frac{e_i}{e_f} = \frac{1}{W_r}\end{aligned}\tag{A.5}$$

Adding (A.3) and (A.4) and using the definition of (A.5) obtain

$$\Delta E = \frac{W_f e_f}{gJ} \left\{ 1 - \frac{W_r}{E_r} + \int_i^f \frac{\frac{W}{W_f}}{\left( \frac{T}{D} - 1 \right)} d\chi \right\}\tag{A.6}$$

One can now relate ( $W$ ) to ( $E$ ) by assuming that the change in energy is due entirely to the combustion of the fuel

$$\eta_p Q = \frac{\Delta E}{\Delta W_{propellant}} \quad (\text{A.7})$$

Then, can define the energy as

$$E = \frac{e}{\eta_p Q J g} \quad (\text{A.8})$$

Using (A.7) and (A.8), one can now rewrite (A.6) as

$$\eta_p Q (W_r - 1) = \frac{e_f}{gJ} \left\{ 1 - \frac{W_r}{E_r} + \int_i^f \frac{\frac{W}{W_f}}{\left( \frac{T}{D} - 1 \right)} d\chi \right\} \quad (\text{A.9})$$

Consider the integral term. First rewrite the numerator of the integrand as

$$\frac{W}{W_f} = (W_r - 1) \left( \frac{1 - \chi}{1 - \chi_i} \right) + 1 \quad (\text{A.10})$$

Formally defining the integral with this definition can, after considerable “massaging” of terms, get the following representation

$$\int_i^f \frac{W}{W_f} d\chi = \left( \frac{W_r + 1}{2} \right) (1 - \chi_i) \quad (\text{A.11})$$

With this definition, can now rewrite (A.9)

$$\eta_p Q (W_r - 1) = \frac{e_f}{gJ} \left\{ 1 - W_r \chi_i + \frac{(1 - \chi_i)^2}{2 \left( \frac{T}{D} - 1 \right)} + \frac{W_r \left( 1 - \frac{\chi_i^2}{2} \right)}{2 \left( \frac{T}{D} - 1 \right)} \right\} \quad (\text{A.12})$$

One can now solve for the fuel weight ratio

$$W_r = \frac{1+E_f \left\{ 1 + \frac{(1-\chi_i)^2}{2 \left( \frac{T}{D} - 1 \right)} \right\}}{1+E_i - E_f \left\{ \frac{\left( 1 - \frac{\chi_i^2}{2} \right)}{2 \left( \frac{T}{D} - 1 \right)} \right\}} \quad (\text{A.13})$$

The expression in (A.13) is the definition of the fuel weight fraction ( $\beta$ ), which is used in Chapter III.

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